Homework

1) We know that the formula for Büchi games is:

$$\langle 1 \rangle \Box \Diamond B = \nu Y. \mu X. \begin{pmatrix} B \cap Pre_1(Y) \\ \cup \\ \neg B \cap Pre_1(X) \end{pmatrix} \qquad (1)$$

Consider the alternative formula:

$$\nu Y. \left(Pre_1(Y) \cap \mu X. \left(B \cup Pre_1(X) \right) \right) \qquad (2)$$

Question: Is any of the following statements true?

(a) On all 2-player games, (1) $\subseteq$ (2).

(b) " " " " , (2) $\subseteq$ (1).

Question

Give an example of a state that belongs to (1) but not to (2), or vice-versa.

2) We know that the solution formula for co Büchi games is:

$$\langle 1 \rangle \Diamond \Box B = \mu X . \nu Y . \begin{pmatrix} \neg B \cap Pre_1(X) \\ B \cap Pre_1(Y) \end{pmatrix} = \varphi_1$$

Consider

$$\varphi_2 = \mu X . \left(\nu Y . \left(B \cap Pre_1(Y) \right) \cup Pre_1(X) \right)$$

(a) Is it true that on all games (choose one):

$$\varphi_1 \subseteq \varphi_2 \quad \text{or} \quad \varphi_2 \subseteq \varphi_1 \quad ?$$

Explain.

(b) Give an example of a game where the two formulas compute different sets of states.

(c) What is the complexity (the time) of computing φ_1 on a game? What about φ_2?

(d) Are φ_1 and φ_2 different on games where pl. 1 plays at all states? (So pl. 2 never plays)? Explain.