Lecture 1

Nash Equilibria:

Definitions and Examples.

Strategic Game: $\langle N, A, u \rangle$

- N: set of players
- For $i \in N$: A_i is the set of actions (also called moves) of player i.
- $u_i : \times_{i \in N} A_i \longrightarrow \mathbb{R}$ ~~preferences~~ utility function.

We let:

$$A = \times_{i \in N} A_i$$

so, if $N = \{1, 2, 3\}$, $a_1 \in A_1$, $a_2 \in A_2$, $a_3 \in A_3$, $(a_1, a_2, a_3) \in A$.

A is the set of "action profiles".

Player $i \in N$ prefers $a \in A$ to $b \in A$ iff

$$u_i(a) \geqslant u_i(b).$$

Graphical representation:

$A_2 = \{1, 2, 3\}$

$A_1 = \{1, 2, 3\}$

	1	2	3
1			
2			
3		2, 5	

$u_1(3,2) = 2$

$u_2(3,2) = 5$

Nash Equilibria

For $i \in N$, and $a \in A$, let a_{-i} be a strategy profile, or action profile, for all the players except i.

Thus, for $a_i \in A_i$, (a_i, a_{-i}) is an action profile for all players, where:

- player i plays a_i
- player j, for $j \neq i$, plays $a_{-i}(j)$.

Nash equilibrium: an action profile a^* such that:

$\forall i \in N: \quad \forall a_i \in A_i:$

$$u_i(a_i, a^*_{-i}) \leqslant u_i(a^*).$$

Single-handedly changing one's strategy does not confer higher utility.

Another way to define Nash equilibria is via the best-response function.

$B_i(a_{-i})$: set of best responses ~~to~~ of player i when the others play a_{-i}.

$$B_i(a_{-i}) = \{ a_i \in A_i \mid u_i(a_i, a_{-i}) \geq u_i(a'_i, a_{-i}) \text{ for all } a'_i \in A_i \}.$$

Nash equilibrium: (alternative definition)

a^* is a Nash equilibrium if, for all $i \in N$,

$$a^*_i \in B_i(a^*_{-i}).$$

Examples:

BoS

	B	S
B	2,1	0,0
S	0,0	1,2

BoS for ancient music lovers

	B	S
B	2,2	0,0
S	0,0	1,1

This game has no Nash equilibrium:

Penny Matching

	0	1
0	1, −1	−1, 1
1	−1, 1	1, −1

Prisoner's Dilemma:

	N	C
N	3,3	0,4
C	4,0	1,1

Problem 1:

Give a ~~ga~~ 2-player game with a Nash equilibrium a^*, such that you can find a_1 and a_2 with:

- $a_1 \in B_1(a_2^*)$
- $a_2 \in B_2(a_1^*)$
- (a_1, a_2) is <u>not</u> a Nash equilibrium.