Mechanism Design.

Environment

$E = (n, C, \mathcal{U})$

- n : n. of players. Note: we assume we know who is playing (see later in auctions).
- C : the set of outcomes.
- $\mathcal{U}$: the set of preferences, or utility functions. Each $u \in \mathcal{U}$ is a vector, where, for $i \in [1, \dots, n]$, u_i is the utility to player i.

Choice rule: $f : \mathcal{U} \mapsto 2^C$

Tells us what are the acceptable outcomes.

Mechanism Design:

Create a game with ~~strategies~~ sets of strategies $X_1, \dots, X_n$, and outcome function g,

$$g : X_1 \times \cdots \times X_n \mapsto C$$

[illegible]
[illegible]

<u>Goal of mechanism design:</u>

The utility to player $i \in [1 \dots n]$ of $x_1, \dots, x_n$ is:

$$u_i(x_1 \dots x_n) = u_i(g(x_1 \dots x_n))$$

- For each player i, there is a weak dominating strategy Dom_i.

$$Dom = \bigtimes_{i=1}^{n} Dom_i.$$

- If $(x_1 \dots x_n) \in Dom$, then

$$g(x_1 \dots x_n) \in f(u_1 \dots u_n).$$

"If players play weakly dominating strategies, the outcome is acceptable".
STRONG IMPLEMENTATION.

- Weak implementation:
There is $(x_1 \dots x_n) \in Dom$ with

$$g(x_1 \dots x_n) \in f(u_1 \dots u_n).$$

Truthful implementation.

An implementation is truthful if the strategy space is $\mathcal{U}_1 \times \cdots \times \mathcal{U}_n$, where

$$\mathcal{U}_i = \{u_i \mid \exists u_{-i} . (u_i, u_{-i}) \in \mathcal{U}\}$$

and for which $(u_i, \ldots, u_n)$ is a dominant strategy in the game with utility $(u_1, \ldots, u_n)$.

<u>Theorem</u> (revelation principle)

If E, f are Dom-implementable, then there is a weak truthful implementation.

<u>Proof</u> Suppose X ~~as~~, g is a Dom-implementation, where $X_i(u_i)$ is the move of player i when his utility is u_i.

Define g^* by:

$$g^*(\mathcal{U}) = g(X_1(u_1), \ldots, X_n(u_n)).$$

Then, $\mathcal{U}, g^*$ is a weak truthful implementation.

Example

Construction of a public good G.

The good G costs C to build.

~~n~~ There are n people, the utility of G to i is u_i.

The good should be built only if $\sum_i u_i \geq C$.

Note that we don't care about recouping C (taxes do that).

How can force the persons to tell the truth?

~~answers~~

~~Th~~ Methods that don't work:

- Have each person pay $\frac{C}{n}$. Then, if $u_i > \frac{C}{n}$, i says it's worth C+1 to her.
- ~~Have~~ ask each person for u_i, and have them pay u_i. They would ~~so~~ under-report their utility.

Method that works:

Make each person i pay

$$\max\left(0,\; C - \sum_{j \neq i} u'_j\right) \quad \text{if } G \text{ is built} \quad (0 \text{ otherwise}).$$

where u'_j is the declared utility of j.

Note (see also later): what person i pays does not depend on what i says. As we will see, this will be an underlying principle.

Proof: Note: i pays $q_i \leq u'_i$.

Case analysis:

1) $u'_i > u_i$. Then, if $\sum_j u'_j > C$, but $\sum_j u_j < C$, the good is built incorrectly. But person i also pays more than u_i.

2) $u'_i < u_i$. The good may be not built if $\sum_j u'_j < C$, even though $\sum_j u_j > C$.

The person pays 0, while declaring u'_i, it might have paid no more than u_i for something worth u_i.