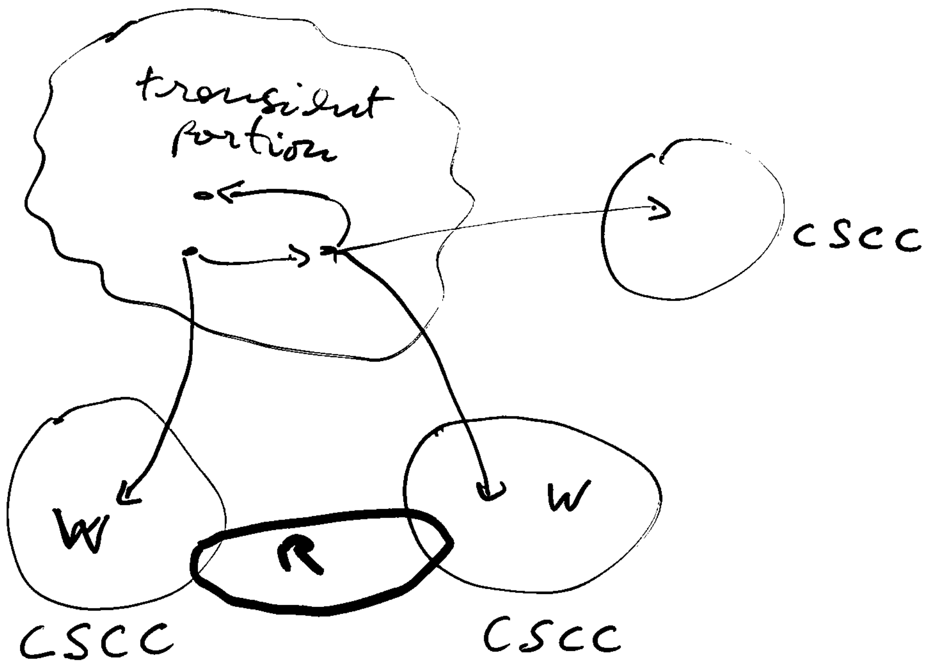
Computing $\Box\Diamond R$ in a MC:

1) Decompose the chain in a transient portion, and the closed s.c. components



We must compute the prob. of reaching the union of the CSCC that $\cap R \neq \emptyset$, call this W.
Let T be the set of trans. states.

Then

$$P = \text{matrix } [P_{st}]_{s,t \in T}$$

$x_{t \in T}$: vector of winning prob.

$s \in T$:

$$x_s = \sum_{t \in T} P_{st} x_t + \sum_{t \in W} \tilde{P}_{st}$$

Let $c_s = \sum_{t \in W} \tilde{p}_{st}$.

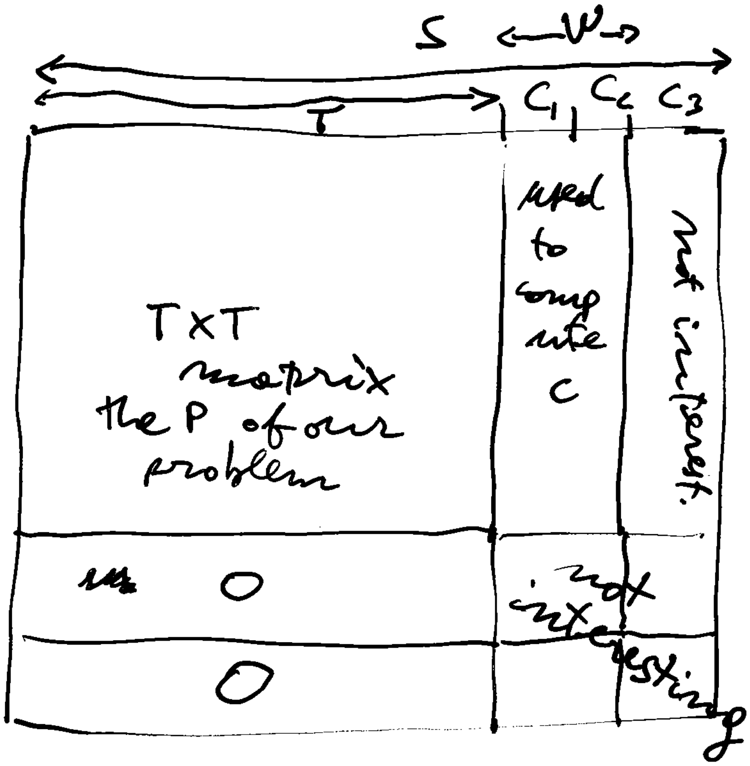
$$x = Px + c$$

where: P is a matrix $[p_{st}]_{s,t \in T}$

c is a constant vector $[c_s]_{s \in T}$

x is a vector $[x_s]_{s \in T}$.

S, P, T, and W:



$$x = Px + c$$

$$Ix = Px + c$$

$$(I-P)x = c$$

$$x = (I-P)^{-1} c \; .$$

$$x = c \setminus (I-P)$$ (Matlab)

$(I-P)^{-1}$ exists, and is unique.

$(I-P)$ is invertible.

Simulation solution.

Idea: to compute ~~X(s)~~ X_s, simulate the MC from s, many times, and look at the prob. of winning.

You get an estimate $\hat{X}_s$ out of the simulations.

Use $\hat{X}_s$, in:

$$\eta_1(s) = \arg\max_{a \in \Gamma(s)} \sum_t \tau(s,a)(t) \cdot \hat{X}(t).$$

Non-Zero Sum Game

Γ_1, Γ_2: sets of moves

$\Gamma_2 = \{1, 2, 3\}$

$\Gamma_1 = \{1, 2, 3\}$

	1	2	3
1			
2			
3		2, 5	

$u_1(3,2)$: reward to pl. 1 when
1 plays 3
2 plays 2

$u_2(3,2)$ reward to pl. 2

Zero-sum if,

$$\forall a \in \Gamma_1, \forall b \in \Gamma_2, \quad u_1(a,b) = -u_2(a,b)$$

$(u_1(a,b) = 1 - u_2(a,b)$, but it's the same$)$

<u>Notation</u> : π_i: strategy (move) for pl. i
(non random for now)

a_i: move of pl. i

π_{-i}: strategy for all others.

If players = $\{1, 2, 3\}$, $\pi_{-1} = (\pi_2, \pi_3)$.

Nash Equilibrium: a strategy tuple $(\pi_1^* \dots \pi_n^*)$ (n = n. of players) such that:

$\forall i \in 1 \dots n$,

$$(\pi_i^*, \pi_{-i}^*) \underbrace{\geq_i}_{\text{pl. } i \text{ prefers}} (\pi_i, \pi_{-i}^*) \quad \forall \pi_i$$

$$u_i(\pi_i^*, \pi_{-i}^*) \geq u_i(\pi_i, \pi_{-i}^*)$$

Best Response $B_i(\pi_{-i})$:

$\cancel{B_i(\pi_{-i}) = \{\pi_i \in \Pi_i \mid (\pi_i, \pi_{-i})}$

$B_i(\pi_{-i}) = \{\pi_i \mid u_i(\pi_i, \pi_{-i}) \geq u_i(\pi_i', \pi_{-i}) \text{ for all } \pi_i'\}$

Theorem $(\pi_1^*, \ldots, \pi_n^*)$ is a Nash equil

iff

$$\pi_i^* \in B_i(\pi_{-i}^*) \quad \forall i \in 1..n.$$

EXAMPLES

BoS

	B	S
B	2,1	0,0
S	0,0	1,2

Bach
Stravinsky

Modern
Ancient

	B	S
B	2,2	0,0
S	0,0	1,1

Two Ancient

	0	1
0	1,-1	-1,1
1	-1,1	1,-1

No Nash
(Nash if randomized)

Prisoner

1 \ 2	N	C
N	3,3	0,4
C	4,0	1,1

Hawk-Dove

	D	H
D	3,3	1,4
H	4,1	0,0