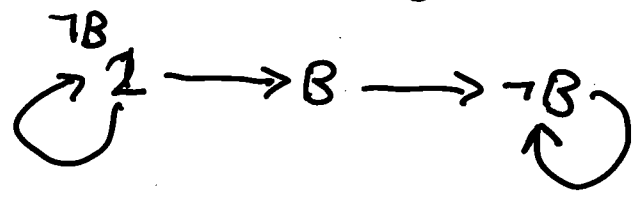
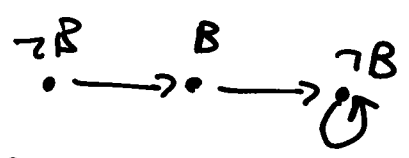
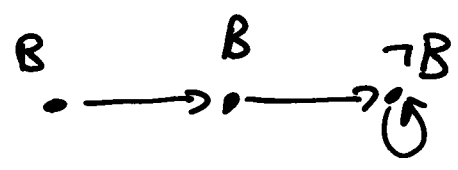


$$\forall Y. \mu X. \left(\begin{array}{c} \neg B \wedge \text{Pre}_1(X) \\ \vee \\ B \wedge \text{Pre}_1(Y) \end{array} \right)$$

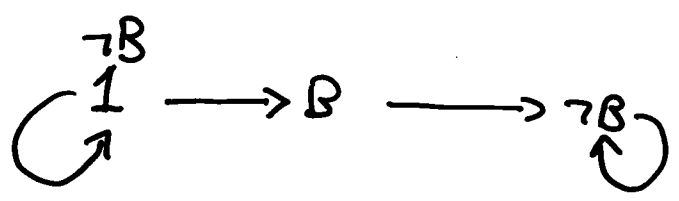
$\left. \begin{array}{l} \\ \\ \end{array} \right\} \neg$
 $\subseteq \langle 1 \rangle \square \diamond B$
 Thm A

$$\mu Y. \forall X. \left(\begin{array}{c} \neg B \wedge \text{Pre}_2(X) \\ \vee \\ B \wedge \text{Pre}_2(Y) \end{array} \right)$$

$\subseteq \langle 2 \rangle \square \square \neg B.$
 & Proof Thm B



all states are in $\langle 2 \rangle \square \square \neg B.$

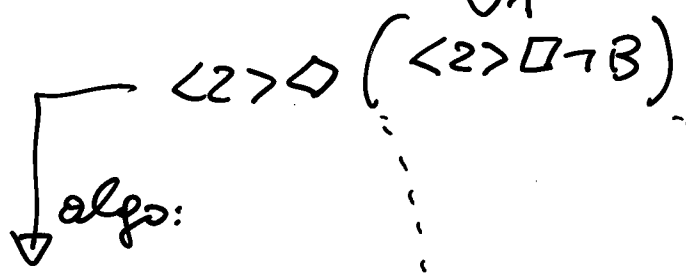


"You don't know when eternity has arrived"

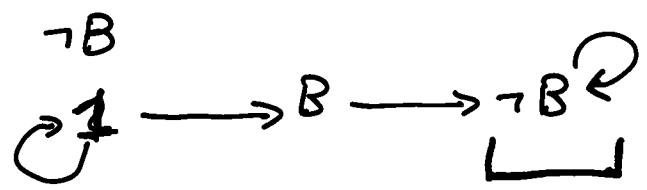
Wrong, naïve algorithms for

$$\langle 2 \rangle \diamond \square \neg B$$

$\vee X$



$$\mu Y. (\vee X. (\neg B \wedge \text{Pre}_2(X)) \vee \text{Pre}_2(Y))$$



$$\langle 2 \rangle \square \neg B$$

$$\langle 2 \rangle \diamond (\langle 2 \rangle \square \neg B)$$

$$\langle 2 \rangle \diamond \square \neg B$$

$$\text{temp} = \vee X. (\neg B \wedge \text{Pre}_2(X))$$

$$\vee Y. (\text{temp} \vee \text{Pre}_2(Y))$$

Thm: If only player 2 can choose,
then

$$\langle 2 \rangle \diamond \square \neg B = \langle 2 \rangle \diamond \langle 2 \rangle \square \neg B$$

(HW)

A μ -calculus formula φ is alternation-free iff, every

subformula of the type

$$\mu x. \gamma(x)$$

$$\nu x. \gamma(x)$$

is such that γ has no other free variable than x .

Then (...) an alternation-free μ -calculus formula⁴ can be computed with a linear number of iterations,

$$\hat{\quad} \text{in } |S|$$

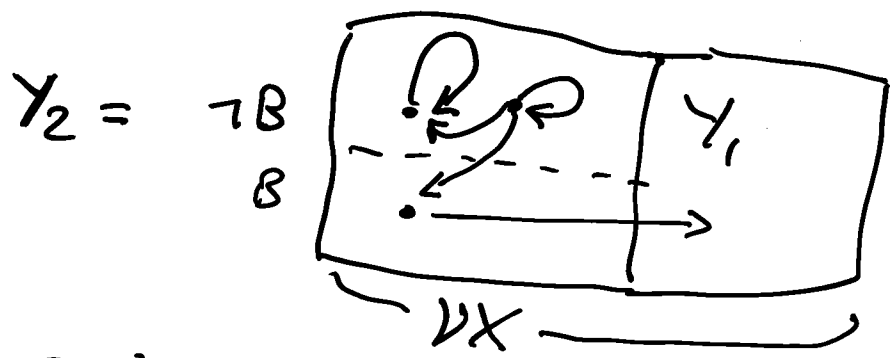
and in $O(|S| \cdot |\varphi|)$ iterations.

$$\mu Y. \forall X. \left(\begin{array}{c} \neg B \wedge \text{Pres}(X) \\ \vee \\ B \wedge \text{Pres}(Y) \end{array} \right)$$

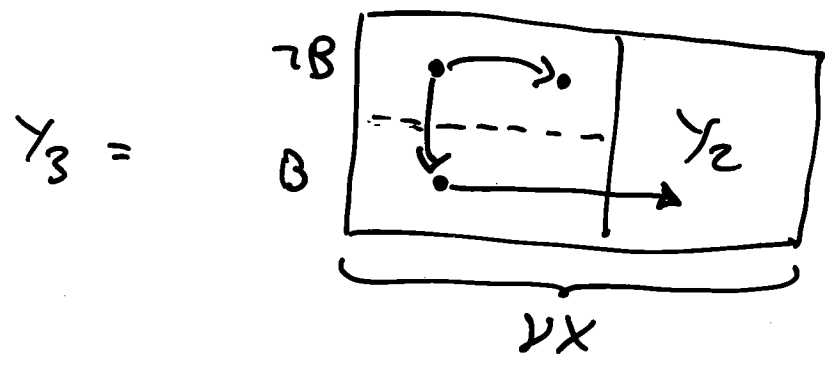
$$Y_0 = \emptyset$$

$$Y_1 = \forall X. \left(\begin{array}{c} \neg B \wedge \text{Pres}(X) \\ \vee \\ \cancel{B \wedge \text{Pres}(\emptyset)} \end{array} \right) = \langle 2 \rangle \square \neg B.$$

$$Y_2 = \forall X. \left(\begin{array}{c} \neg B \wedge \text{Pres}(X) \\ \vee \\ B \wedge \text{Pres}(Y_1) \end{array} \right)$$



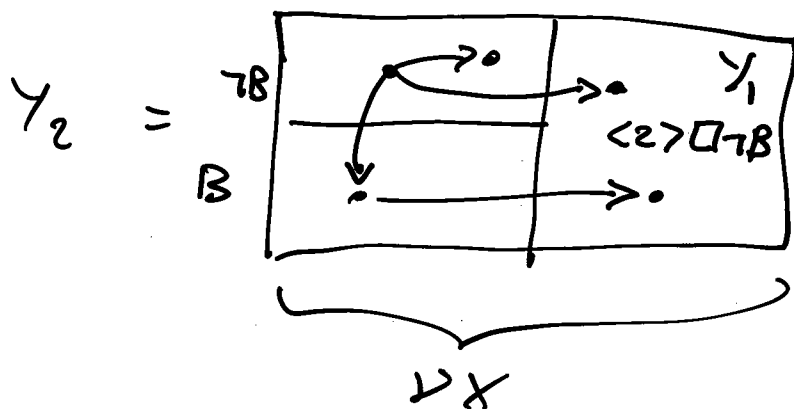
$Y_2 = \langle 2 \rangle$ touch B at most once.



$$Y_1 = \forall X. (\neg B \wedge \text{Pres}_2(X))$$

$$Y_1 = \boxed{\begin{array}{c} \neg B \\ \circ \rightarrow \circ \end{array}} = \langle 2 \rangle \square \neg B$$

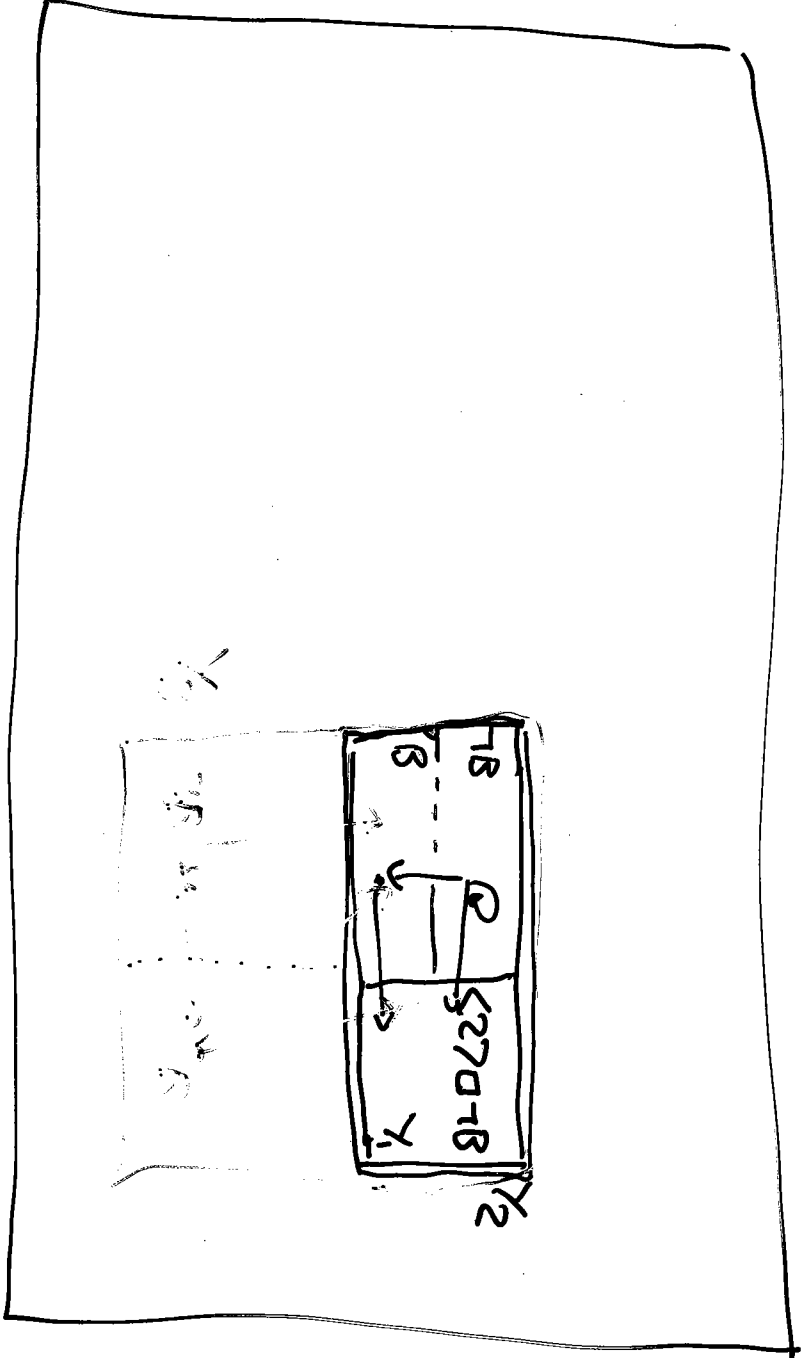
$$Y_2 = \forall X. \left(\begin{array}{c} B \wedge \text{Pres}_2(Y_1) \\ \vee \\ \neg B \wedge \text{Pres}_2(X) \end{array} \right)$$

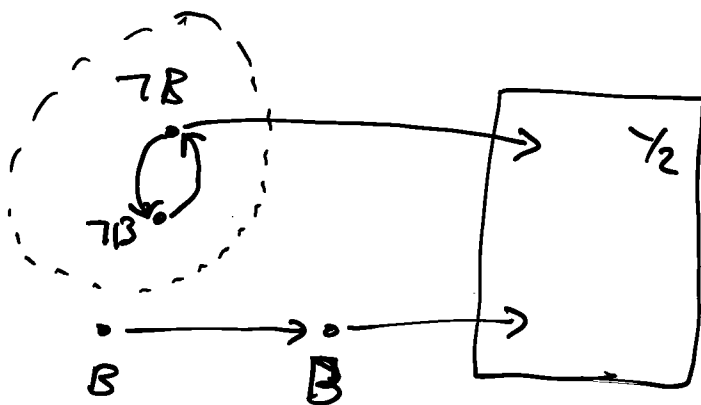


$$Y_3 = \underbrace{\begin{array}{|c|c|} \hline \begin{array}{c} \neg B \\ \circ \end{array} & \begin{array}{c} Y_2 \\ \circ \end{array} \\ \hline \begin{array}{c} B \\ \circ \end{array} & \begin{array}{c} \circ \end{array} \\ \hline \end{array}}_{\forall X} \text{ at most once in } B$$

The diagram shows a table with two columns and two rows. The left column is labeled with $\neg B$ at the top and B at the bottom. The right column is labeled with Y_2 at the top and \circ at the bottom. In the top-left cell, there is a dot with an arrow pointing to the right. In the top-right cell, there is a dot with an arrow pointing to the left. In the bottom-left cell, there is a dot with an arrow pointing to the right. In the bottom-right cell, there is a dot with an arrow pointing to the right. A large curly brace under the entire table is labeled $\forall X$. A diagonal slash is drawn through the right column, and the text "at most once in B" is written to the right of the table.

= at most twice in B.





So, γ_k pl. 2 from γ_k can ensure to touch B at most $k-1$ times.

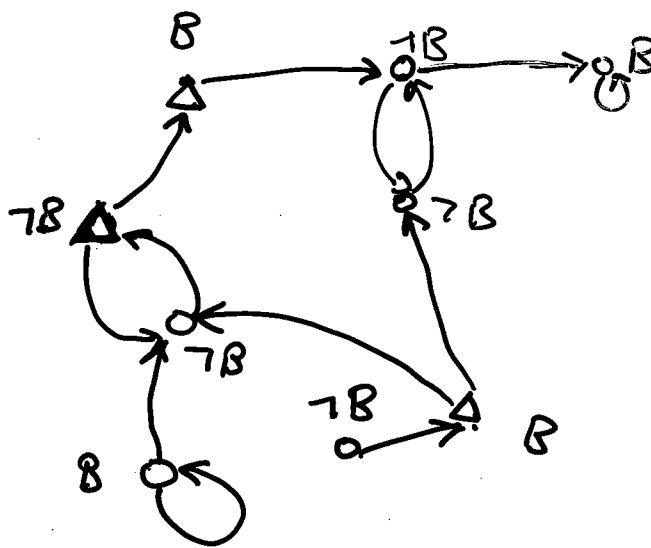
So, pl. 2 from $\mu \gamma_0$ (---) can ensure that B is touched at most a finite number of times.

So, $\gamma \gamma_0$ (...) $\in \langle \gamma \rangle \diamond \square \gamma B$.

(Thm B).

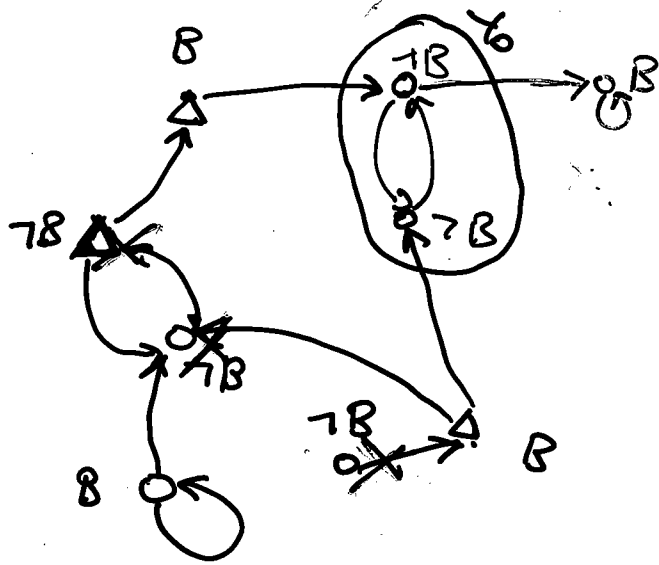
HW: Show why B_{ii} , ωB_u fail on ω games.

$0 = \text{pl. 2}$
 ~~Δ~~ $\Delta = \text{pl. 1}$



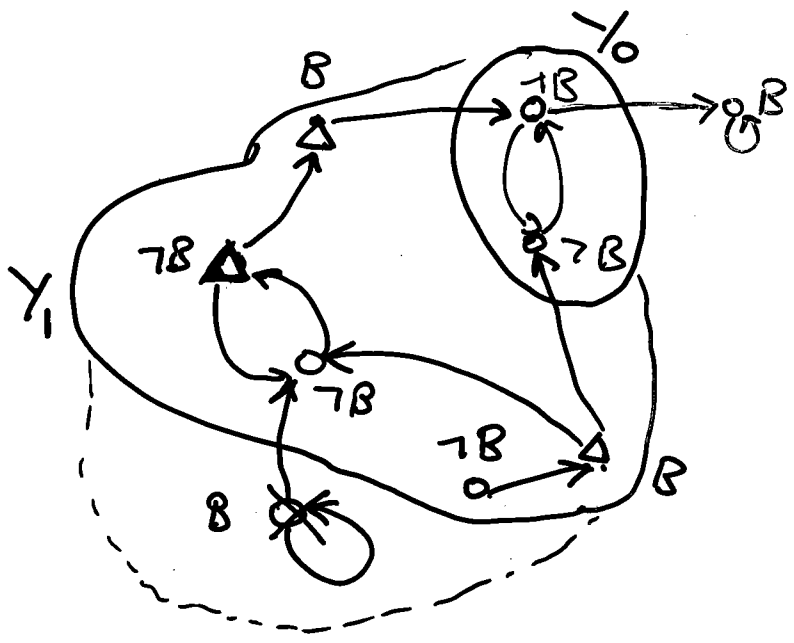
$$Y_0 = \forall X. (\neg B \wedge \text{Pre}_2(X))$$

$0 = pl. 2$
 ~~Δ~~ $= pl. 1$



$$Y_0 = \forall X. (\neg B \wedge Pre_2(X))$$

8
80V2

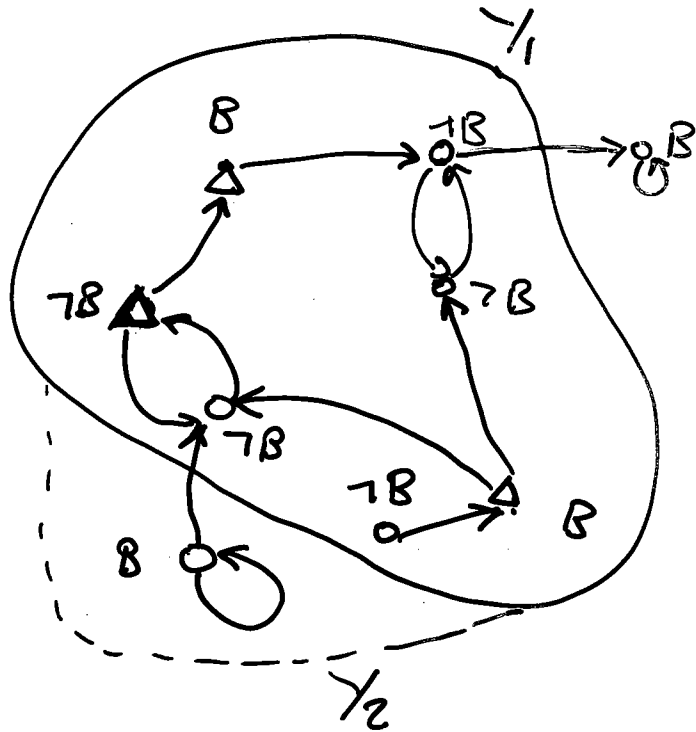


$\circ = pl. 2$
 $\triangle = pl. 1$

$$Y_0 = \forall X. (\neg B \wedge Pre_2(X))$$

$$Y_1 = \forall X. \left(\begin{array}{c} \neg B \wedge Pre_2(X) \\ \vee \\ B \wedge Pre_2(Y_0) \end{array} \right)$$

$0 = pl. 2$
 ~~Δ~~ $\Delta = pl. 1$



$$\gamma_0 = \forall x. (\neg B \wedge \text{Pre}_2(x))$$

$$\gamma_2 = \forall x. \left(\begin{array}{c} \neg B \wedge \text{Pre}_2(x) \\ \vee \\ B \wedge \text{Pre}_2(\gamma_1) \end{array} \right)$$

Regular languages $\subseteq \Sigma^*$



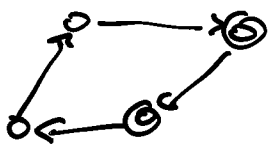
ω -regular $\subseteq \Sigma^\omega$

Regular

- DFA = NFA = regexp
- canonical/minimal DFA.

ω -Regular

(D/N) Büchi automaton



$B \subseteq S$ accepting states.
 a path is accepting
 if B visited ∞ often.

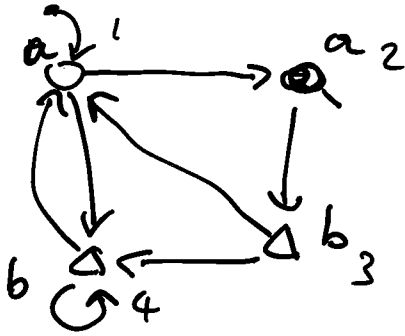
Thm

nondet Büchi automata = ω -reg languages.

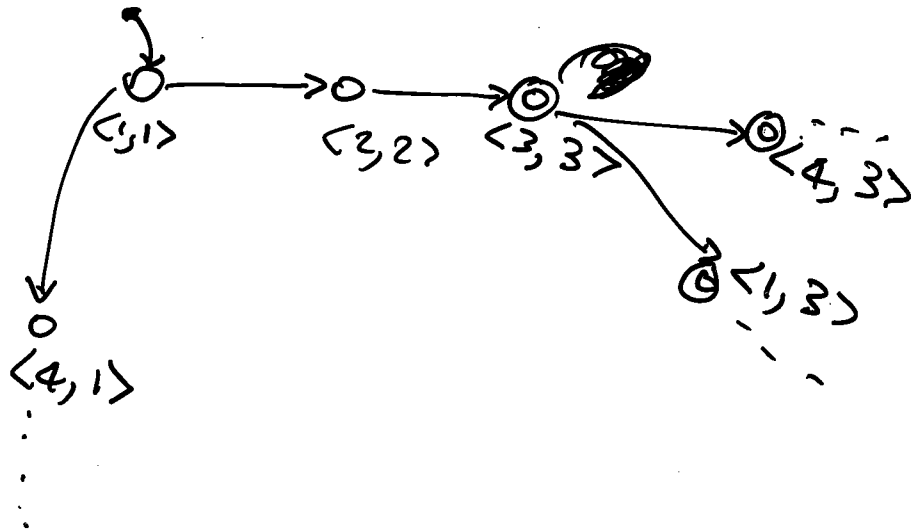
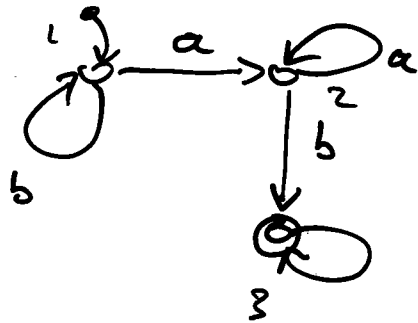
\mathcal{U} (closed under \cup, \cap, \neg, \dots)

det Büchi

Game



Let Automaton



= game annotated with winning cond of let automaton.

det Bild $\not\subseteq$ mondet Bild

$$\Sigma = \{a, b\}$$

$$\square \diamond a$$

\rightsquigarrow

$$\Sigma = \{a, b, c\}$$

$$\square a \vee \diamond b$$

mondet

