$$\nu Y.\ \mu X.\ \begin{pmatrix} \neg B \wedge Pre_1(X) \\ \vee \\ B \wedge Pre_1(Y) \end{pmatrix}$$

$\subseteq \langle 1 \rangle \Box \Diamond B$

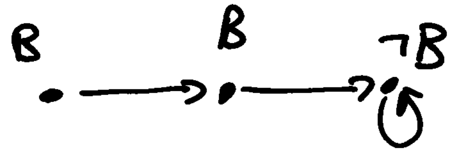
Thm A

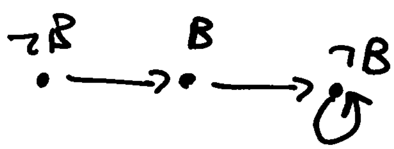
$\neg$

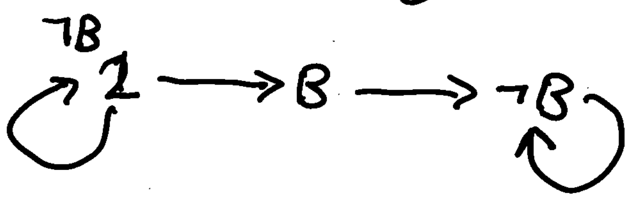
$$\mu Y.\ \nu X.\ \begin{pmatrix} \neg B \wedge Pre_2(X) \\ \vee \\ B \wedge Pre_2(Y) \end{pmatrix}$$

$\subseteq \langle 2 \rangle \Diamond \Box \neg B.$

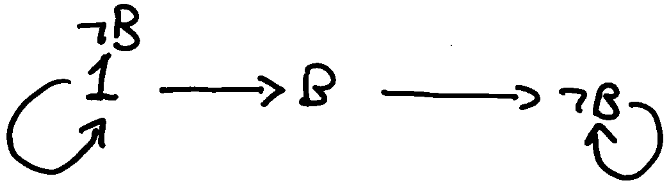
Proof <u>Thm B</u>







all states are in $\langle 2 \rangle \Diamond \Box \neg B$.



"You don't know when eternity has arrived"

Wrong, naive algorithms for

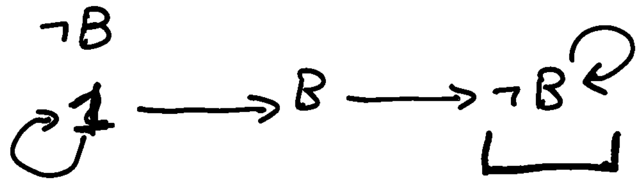
$$\langle 2\rangle \Diamond \Box \neg B$$

$$\cup\!\!\!\times$$

$$\langle 2\rangle \Diamond \left(\langle 2\rangle \Box \neg B \right)$$

algo:

$$\mu Y. \left(\nu X. (\neg B \wedge Pre_2(X)) \vee Pre_2(Y) \right)$$



$\langle 2\rangle \Box \neg B$

$\langle 2\rangle \Diamond \left(\langle 2\rangle \Box \neg B \right)$

$\langle 2\rangle \Diamond \Box \neg B$

$$temp = \nu X. (\neg B \wedge Pre_2(X))$$

$$\nu Y. (temp \vee Pre_2(Y))$$

<u>Thm</u>: If only player 2 can choose, then

$$\langle 2\rangle \Diamond \Box \neg B = \langle 2\rangle \Diamond \langle 2\rangle \Box \neg B$$

(HW)

A μ-calculus formula φ is <u>alternation-free</u> iff, every subformula of the type

$$\mu x.\ \gamma(x)$$
$$\nu x.\ \gamma(x)$$

is such that γ has no other free variable than x.

<u>Thm</u> (...) an alternation-free μ-calculus formula φ can be computed with a linear number of iterations, in $|S|$

and in $O(|S| \cdot |\varphi|)$ iterations.

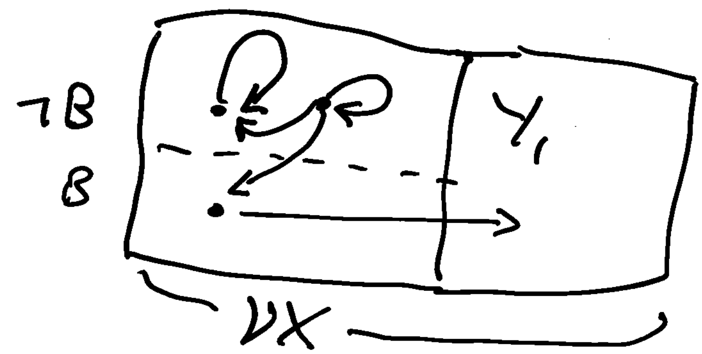
$$\mu Y . \nu X . \begin{pmatrix} \neg B \wedge Pre_2(X) \\ \vee \\ B \wedge Pre_2(Y) \end{pmatrix}$$

$$Y_0 = \emptyset$$

$$Y_1 = \nu X . \begin{pmatrix} \neg B \wedge Pre_2(X) \\ \vee \\ \cancel{B \wedge Pre_2(\emptyset)} \end{pmatrix} = \langle 2 \rangle \Box \neg B .$$

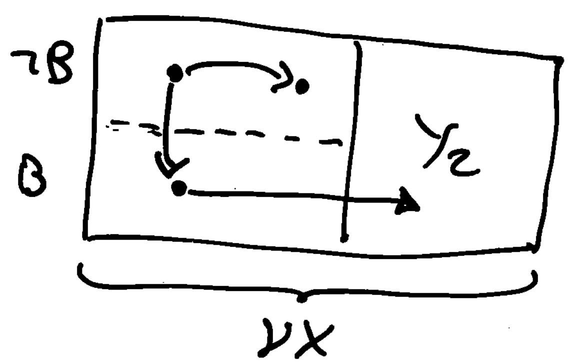
$$Y_2 = \nu X . \begin{pmatrix} \neg B \wedge Pre_2(X) \\ \vee \\ B \wedge Pre_2(Y_1) \end{pmatrix}$$

$Y_2 =$



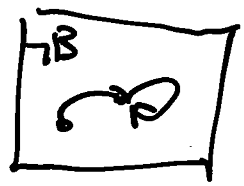
$Y_2 = \langle 2 \rangle$ touch B at most once.

$Y_3 =$



$$Y_1 = \nu X.\,(\neg B \wedge \mathrm{Pre}_2(X))$$

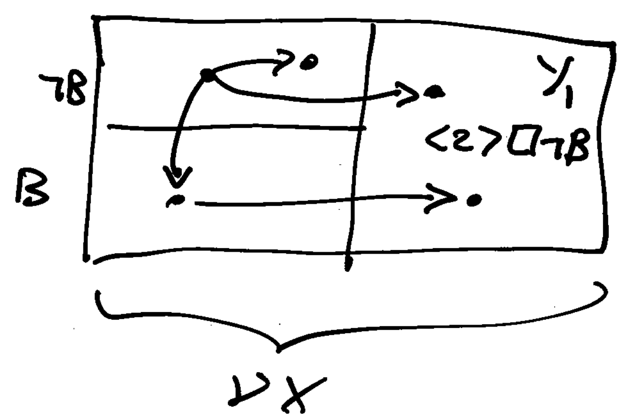
$Y_1 =$



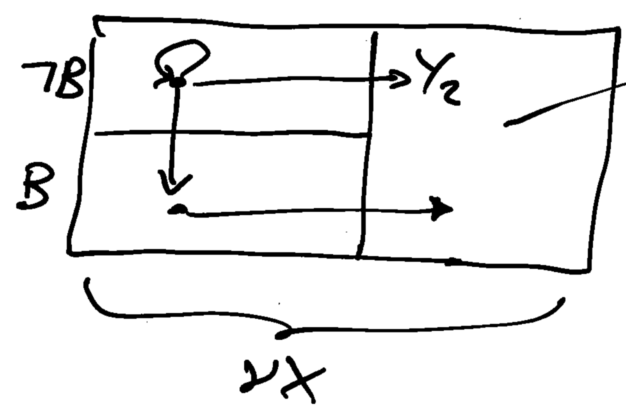
$= \langle 2 \rangle \Box \neg B$

$$Y_2 = \nu X. \begin{pmatrix} B \wedge \mathrm{Pre}_2(Y_1) \\ \vee \\ \neg B \wedge \mathrm{Pre}_2(X) \end{pmatrix}$$

$Y_2 =$

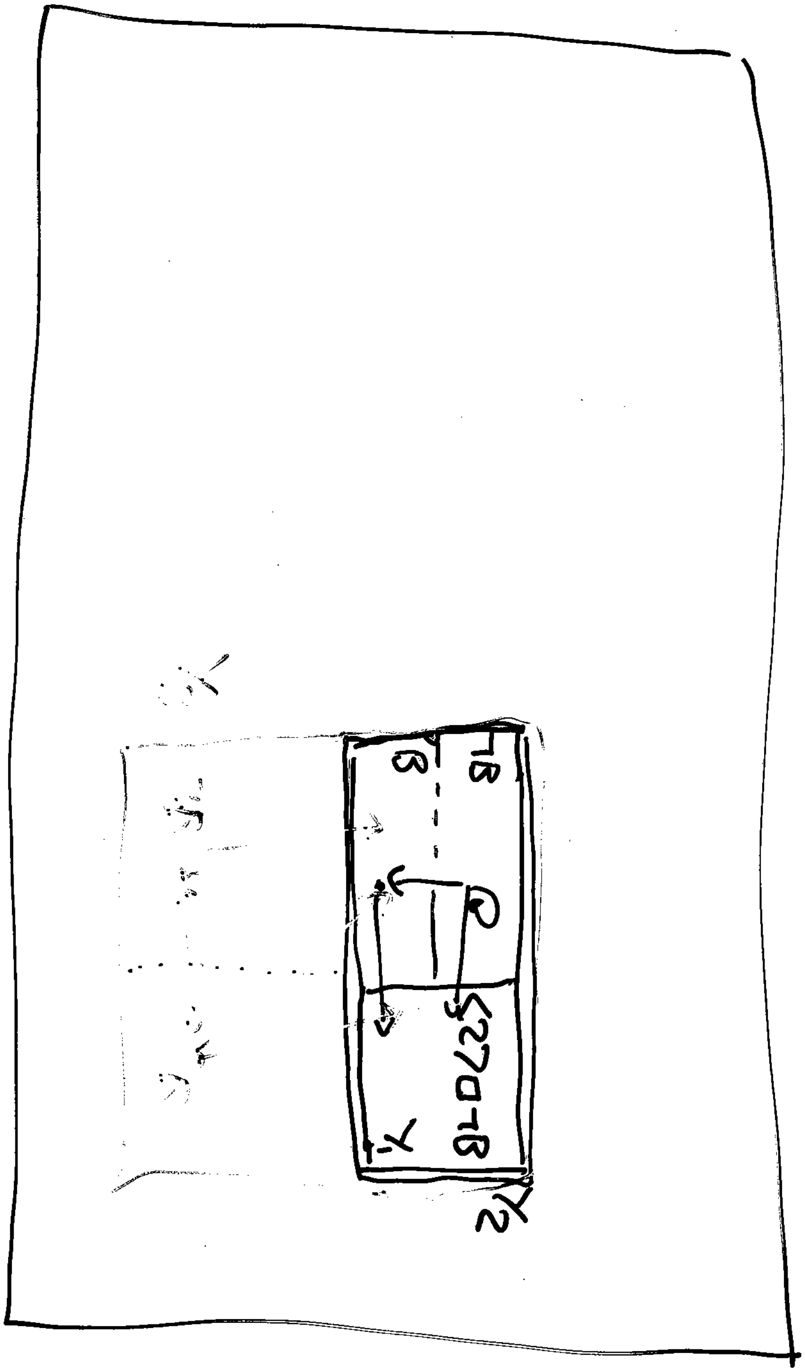


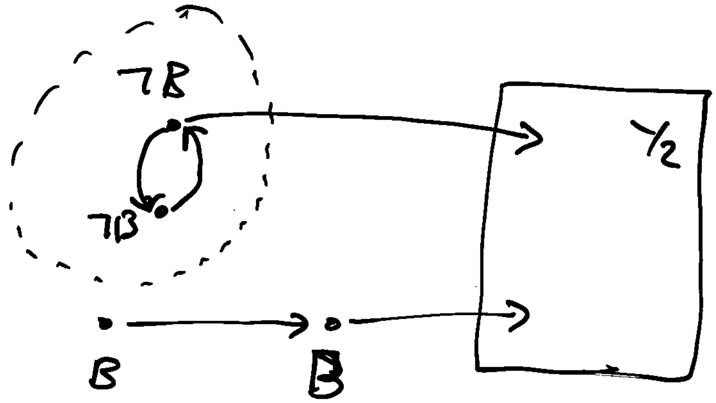
$Y_3 =$



at most once in B

= at most twice in B.


¬B
B
S270¬B
Y1
Y2



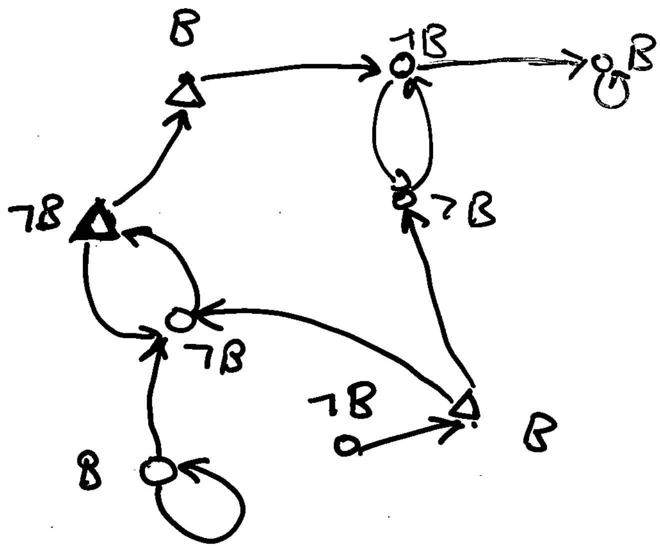
So, ~~Y_k~~ pl. 2 from Y_k can ensure to touch B at most k−1 times.

So, pl. 2 from $\mu Y_0 (\ldots)$ can ensure that B is touched at most a finite number of times.

So, $\nu Y. (\ldots) \subseteq \langle 2 \rangle \Diamond \Box \neg B$.

(Thm B).

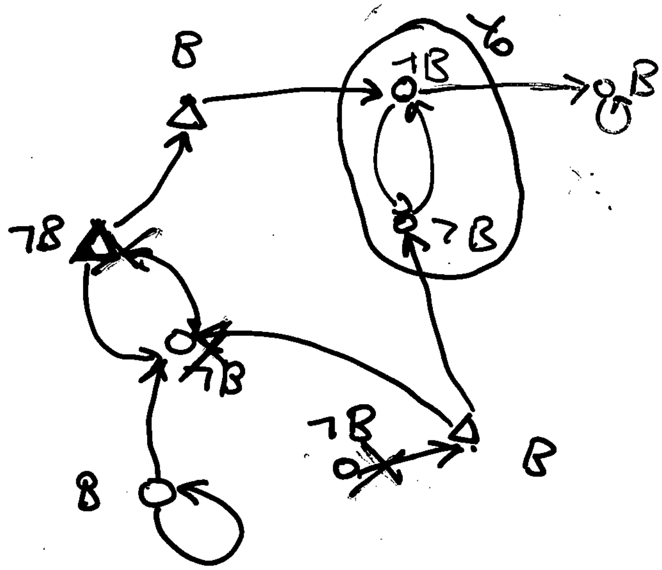
HW: Show why Bii, oBii fail on ∞ games.



○ = pl. 2

△ = pl. 1

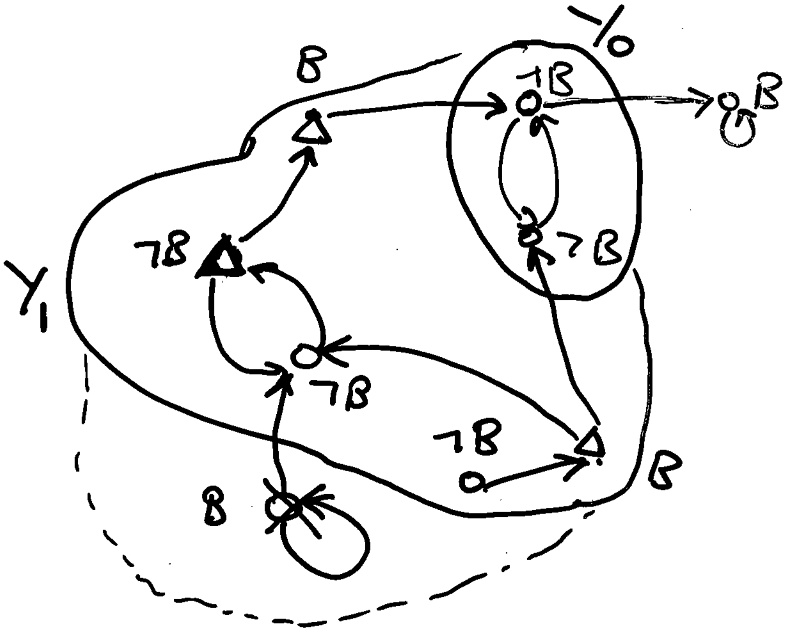
$$Y_0 = \nu X. \left(\neg B \wedge Pre_2(X) \right)$$



$\circ$ = pl. 2

Δ = pl. 1

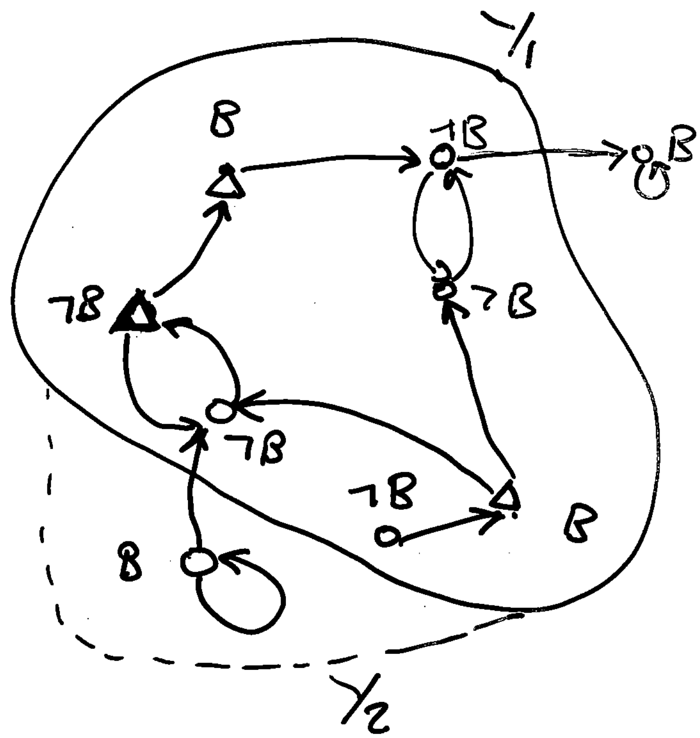
$$Y_0 = \nu X.\left(\neg B \wedge \mathrm{Pre}_2(X)\right)$$



o = pl. 2
Δ = pl. 1

$$Y_0 = \nu X. \left(\neg B \wedge \mathrm{Pre}_2(X) \right)$$

$$Y_1 = \nu X. \left(\begin{array}{c} \neg B \wedge \mathrm{Pre}_2(X) \\ \vee \\ B \wedge \mathrm{Pre}_2(Y_0) \end{array} \right)$$



○ = pl. 2

△ = pl. 1

$$Y_0 = \nu X.\left(\neg B \wedge \mathrm{Pre}_2(X)\right)$$

$$Y_2 = \nu X.\begin{pmatrix} \neg B \wedge \mathrm{Pre}_2(X) \\ \vee \\ B \wedge \mathrm{Pre}_2(Y_1) \end{pmatrix}$$

Regular languages $\subseteq \Sigma^*$

$\downarrow$

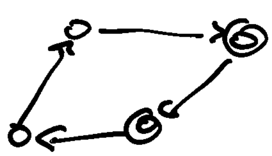
ω-regular $\subseteq \Sigma^\omega$

<u>Regular</u>

- DFA = NFA = regexp
- canonical/minimal DFA.

<u>ω-Regular</u>

$\binom{D}{N}$ Büchi automaton



$B \subseteq S$ accepting states.
a path is accepting if B visited ∞ often.

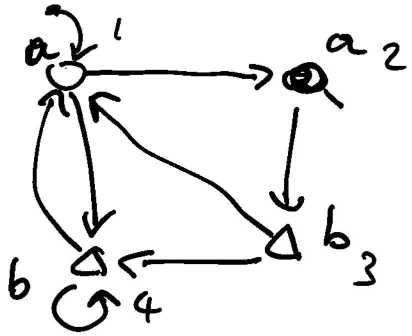
<u>Thm</u>

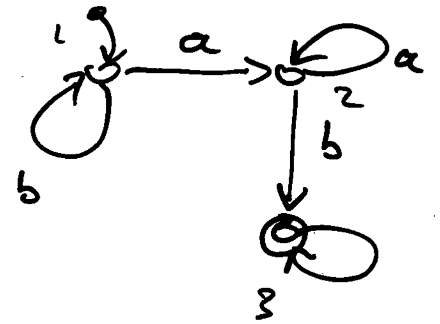
nondet Büchi automata = ω-reg languages.
CH (closed under $\cup, \cap, \neg, \ldots$)

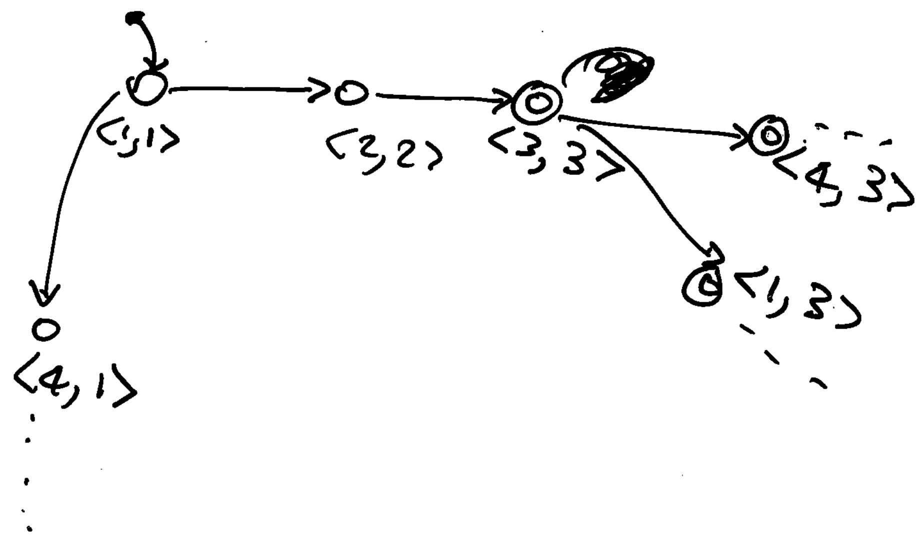
det Büchi

Game

× det automaton







= game annotated with winning cond of det automaton.

det Büchi $\subsetneq$ nondet Büchi

$\Sigma = \{a, b\}$.

$\Box \Diamond a$.

$\Sigma = \{a, b, c\}$

$\Box a \vee \Diamond b$

nondet