Probabilistic Reachability:

$X_0 = \mathbf{0}$ (ds.0) $\left(X_i : S \mapsto [0,1] \right.$
$\left. X_i \in \mathcal{V} \text{ "valuation"} \right)$

$X_{k+1} = [R] \sqcup QPre(X_k)$

pointwise max

If $f, g \in \mathcal{V}$ so $f, g : S \mapsto [0,1]$

$f \sqcup g : S \mapsto [0,1]$

$(f \sqcup g)(s) = \max \{ f(s), g(s) \}$

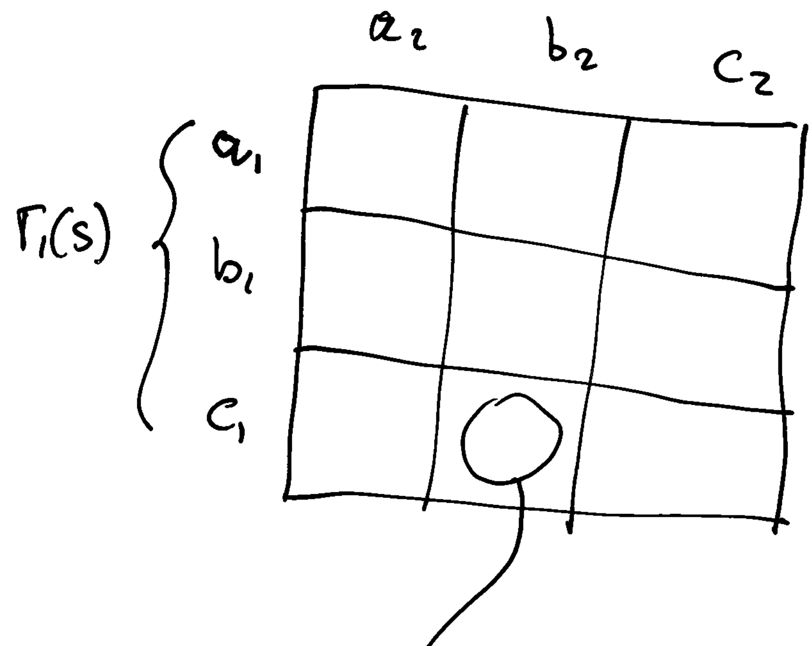
$$[R](s) = \begin{cases} 1 & \text{if } s \in R \\ 0 & \text{if } s \notin R. \end{cases}$$

$$QPre(X)(s) = \overbrace{\sup_{\xi_1 \in D_1(s)} \inf_{\xi_2 \in D_2(s)} E_s^{\xi_1, \xi_2}(X)}^{\text{matrix game}}$$

$\xi_1 \in Distr(\Gamma_1(s))$

ξ_1, ξ_2 : "mixed moved".

to compute $QPre(X)(s)$, consider

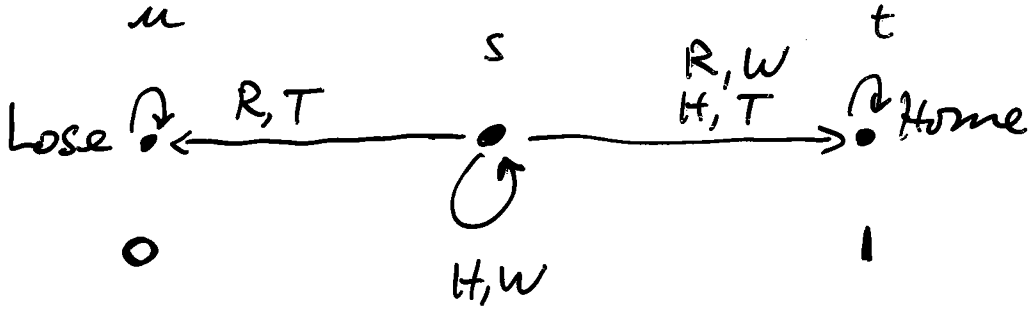


$$U(c_1, b_2) = E_s^{c_1 b_2}(X) = \sum_{t \in S} \delta(s, c_1, b_2)(t) \cdot X(t).$$

$$QPre(X)(s) = \max_{\xi_1 \in Distr(\Gamma_1(s))} \; \min_{\xi_2 \in Distr(\Gamma_2(s))}$$

$$\sum_{\substack{a_1, a_2 \in \Gamma_2(s) \\ \in \Gamma_1(s)}} E_s^{a_1, a_2}(X) \cdot \xi_1(a_1) \cdot \xi_2(a_2).$$

$$= \max_{\xi_1} \inf_{\xi_2} E^{\xi_1 \xi_2}(U)$$



$X(u) = 0$ $\quad$ $X(s) = 9$ $\quad$ $X(t) = 1$

I want $QPre(X)$.

- $QPre(X)(t) = 1$. $\quad$ $QPre(X)(u) = 0$.

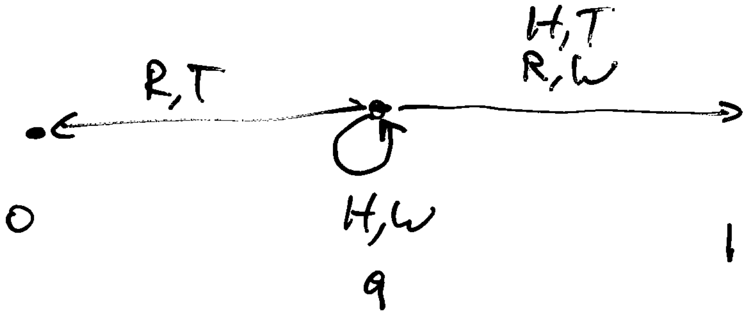
$QPre(X)(s) = ?$ Let us compute it.

What are $\xi_1 \in D_1(s)$?

$$\xi_1 \text{ is: } \begin{array}{cc} R & \alpha \\ H & 1-\alpha \end{array} \quad \text{for } \alpha \in [0,1].$$

$$\max_{\xi_1} \min_{\xi_2} E_s^{\xi_1 \xi_2}(X).$$

If pl. 1 plays ξ_1, pl. 2 can play (as ξ_2) either T, or W, or a combination $\beta T + (1-\beta) W$. But, since either T is better than W, or vice versa, we are not interested in $\beta \in (0,1)$. Just look at T, W.

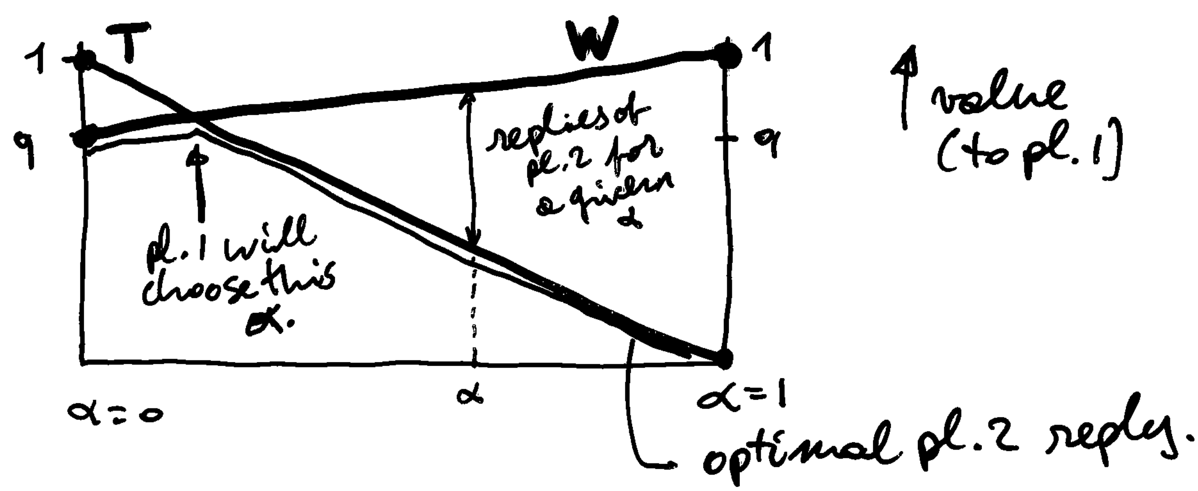


Pl. 1 plays $\alpha R + (1-\alpha) W$.

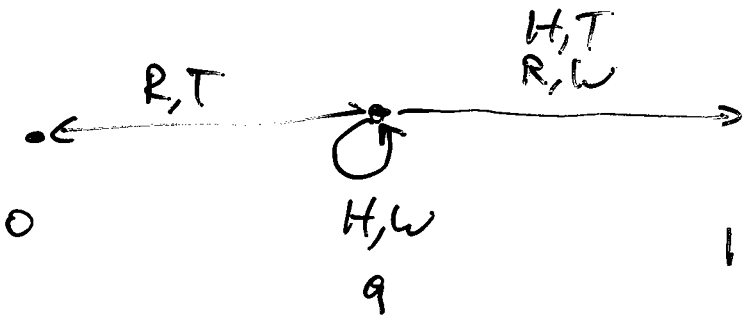
~~Pl 2. plays: T~~

~~revenue $1-\alpha$~~

			Revenue
T:	R	$\alpha \cdot 0$	$1-\alpha$
	H	$1-\alpha \cdot 1$	
W:	R	$\alpha \cdot 1$	$\alpha + (1-\alpha) q$.
	H	$1-\alpha \cdot q$	



$$1-\alpha = \alpha + (1-\alpha) q.$$



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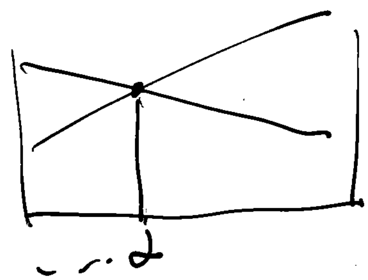
$$1-\alpha = \alpha + (1-\alpha) q .$$

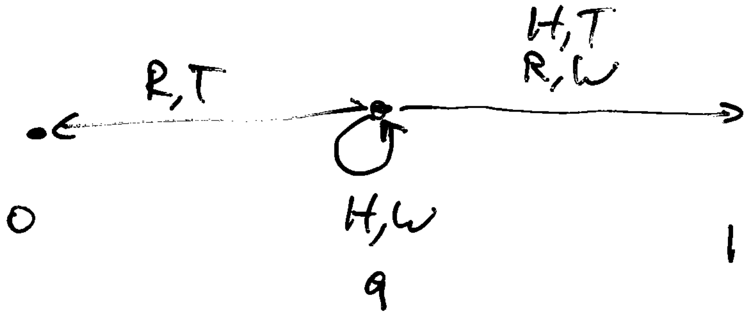
$$\Updownarrow$$

$$\alpha = \frac{1-q}{2-q}$$



$$1-\alpha = \frac{2-q-(1-q)}{2-q} = \frac{1}{2-q} .$$

So, $\mathrm{QPre}(X)(s) = \dfrac{1}{2-q}$, where $q = X(s)$.



$$QPre(X)(s) = \frac{1}{2-q} \qquad q = X(s)$$

$0 \quad 0 \quad 0 \quad = X_0$ / 0 steps.

$0 \quad 0 \quad 1 \quad = X_1 = [R] \circ QPre(X_0)$

$q=0$

$0 \quad \frac{1}{2} \quad 1 \quad = X_2$

$q = \frac{1}{2}$

$1/2-1/2 = \frac{2}{3}$

$0 \quad \frac{2}{3} \quad 1$

$\vdots$

$$\frac{1}{2-\frac{2}{3}} = \frac{3}{6-2} = \frac{3}{4}$$

$0 \quad \frac{i-1}{i} \quad 1 \quad X_i$

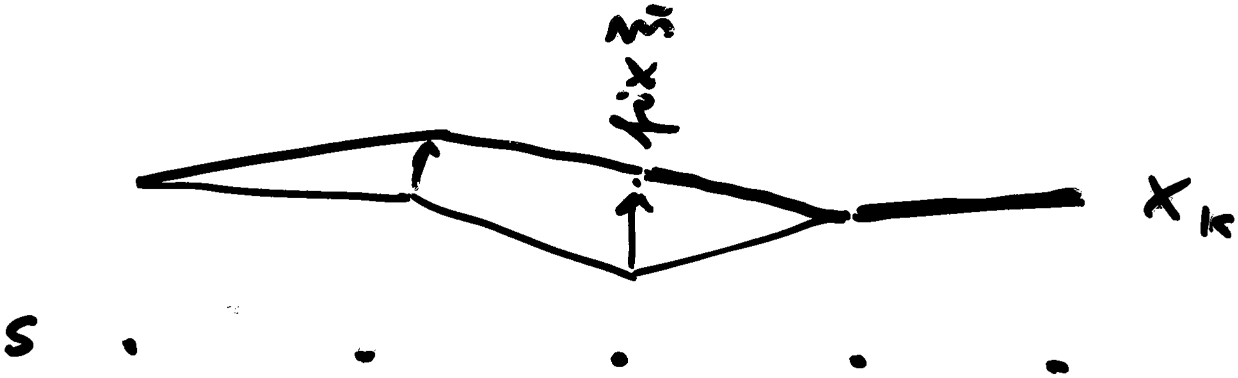
Thm The max prob of going home in at most k steps is $\frac{k}{k+1}$

when you have k rounds to go,

$$q = \frac{k}{k+1}$$

So, at round k+1, use

$$\alpha = \frac{1-q}{2-q} = \frac{\left(1 - \frac{k}{k+1}\right)}{\left(2 - \frac{k}{k+1}\right)}.$$



Give me ε.

I choose k s.t. $|X_k - X_*| \le \varepsilon$.

I run the $\mu X.\,(QPre(X) \sqcup [R])$ algo, $k+1$ times.

For every state s, I remember the ξ_1^s which gave me the last increase in $X_k(s)$.

Memless strategy: at s, play ξ_1^s.